OPTIMAL STRATEGY FOR CONTROLLING THE SEVERITY OF WILDLIFE DISEASE EPIDEMICS DUE TO HARVESTING

Project Advisor: Suzanne Lenhart

Md. Haider Ali Biswas

Suzanne Doig

Moussa Doumbia

Wenrui Hao

Nianpeng Li

Fang Yu

Outline of Talk

- Choisy and Rohani Model
  - Our Simplified Model
- Optimal Control Problem
  - 2 Objective Functionals
  - Model Analysis
  - Model Simulations & Results
- Conclusion

Introduction

- "Harvesting can increase severity of Wildlife disease Epidemics" by *M. Choisy & P. Rohani* (2010)

$$\frac{dS}{dt} = \varphi(t)N(t)B(N) - \left[D_S(N) + \psi(t)q_s H_s + \frac{\beta I(t)}{N(t)} \right] S(t),$$

$$\frac{dE}{dt} = \frac{\beta I(t)S(t)}{N(t)} - \left[D_E(N) + \psi(t)q_E H_E + \sigma\right]E(t),$$

$$\frac{dI}{dt} = \sigma E(t) - \left[D_I(N) + \psi(t)q_I H_I + \gamma + \upsilon\right]I(t),$$

$$\frac{dR}{dt} = \gamma I(t) - \left[D_R(N) + \psi(t)q_R H_R\right]R(t),$$

Introduction

- The results of this paper:
  - Harvesting can increase the disease prevalence and mortality,
  - Interactions between seasonality and D.D. can increase birth rate,
  - Studied the effects of timing on harvesting seasons.

Simplifications

- Assume harvesting effort is identical for all states: $H_i = h(t), i \in \{S, E, I.R\}$
- No seasonality: $\varphi(t) = \psi(t) = 1$
- Successful catchability for all states: $q_i(t) = 1, i \in \{S, E, I, R\}$
- Assume all states have the same per capita death rate: $D_i(N) = d(N),$ $i \in \{S, E, I, R\}$

Simplified Model

- After applying these simplifications our model becomes;

$$\dot{S} = B(N)N(t) - \frac{\beta I(t)S(t)}{N(t)} - (h(t) + d(N))S(t),$$

$$\dot{E} = \frac{\beta I(t)S(t)}{N(t)} - (h(t) + \sigma + d(N))E(t),$$

$$\dot{I} = \sigma E(t) - (h(t) + \upsilon + \gamma + d(N))I(t),$$

$$\dot{R} = \gamma I(t) - (h(t) + d(N))R(t),$$

where $B(N) = b - dN$ and $d(N) = d(1 + N)$.

Optimal Control Problem: 2 Objective Functionals

- We consider two different objective functionals:

$$1^{\text{st}} \text{ Objective Functional}: J = \min \int_0^T \left(\frac{I(t)}{S(t)+E(t)+I(t)+R(t)} + B_1 h^2(t) \right) dt$$

$$2^{\text{nd}} \text{ Objective Functional}: J = \max \int_0^T \big(\alpha(S+R) - (E+I) - B_2 h^2(t) \big) dt$$

- We investigated:
  a) Comparing both O.F. with density dependence
  b) Comparing the affect of density dependence and density independence on the 2^{nd} O.F.

Optimal Control Problem: 2 Cases

- **Case 1:** The 1st objective functional with density dependent birth and death terms.

- **Case 2:** The 2nd objective functional with:
  a) density independent birth and death terms.
  b) density dependent birth and death terms.

Model Analysis: Case One

- **Case 1:** The 1st objective functional with density dependent birth and death.

$$\text{Hamiltonian}: H = \frac{I}{S+E+I+R} + B_1 h^2 + \lambda_S \dot{S} + \lambda_E \dot{E} + \lambda_I \dot{I} + \lambda_R \dot{R}$$

$$0 = \frac{\partial H}{\partial h} = 2B_1 h - \lambda_S S - \lambda_E E - \lambda_I I - \lambda_R R$$

$$\Rightarrow h^* = \frac{\lambda_S S + \lambda_E E + \lambda_I I + \lambda_R R}{2B_1}$$

- Adjoint equations are as follows

Model Analysis: Case One

- Our adjoint equations and transversality conditions are as follows:

$$\lambda_S' = -\frac{\partial H}{\partial S}, \quad \lambda_E' = -\frac{\partial H}{\partial E},$$

$$\lambda_I' = -\frac{\partial H}{\partial I}, \quad \lambda_R' = -\frac{\partial H}{\partial R},$$

$$\lambda_i(T) = 0, \quad i \in \{S, E, I, R\}$$

Model Analysis: Cases Two

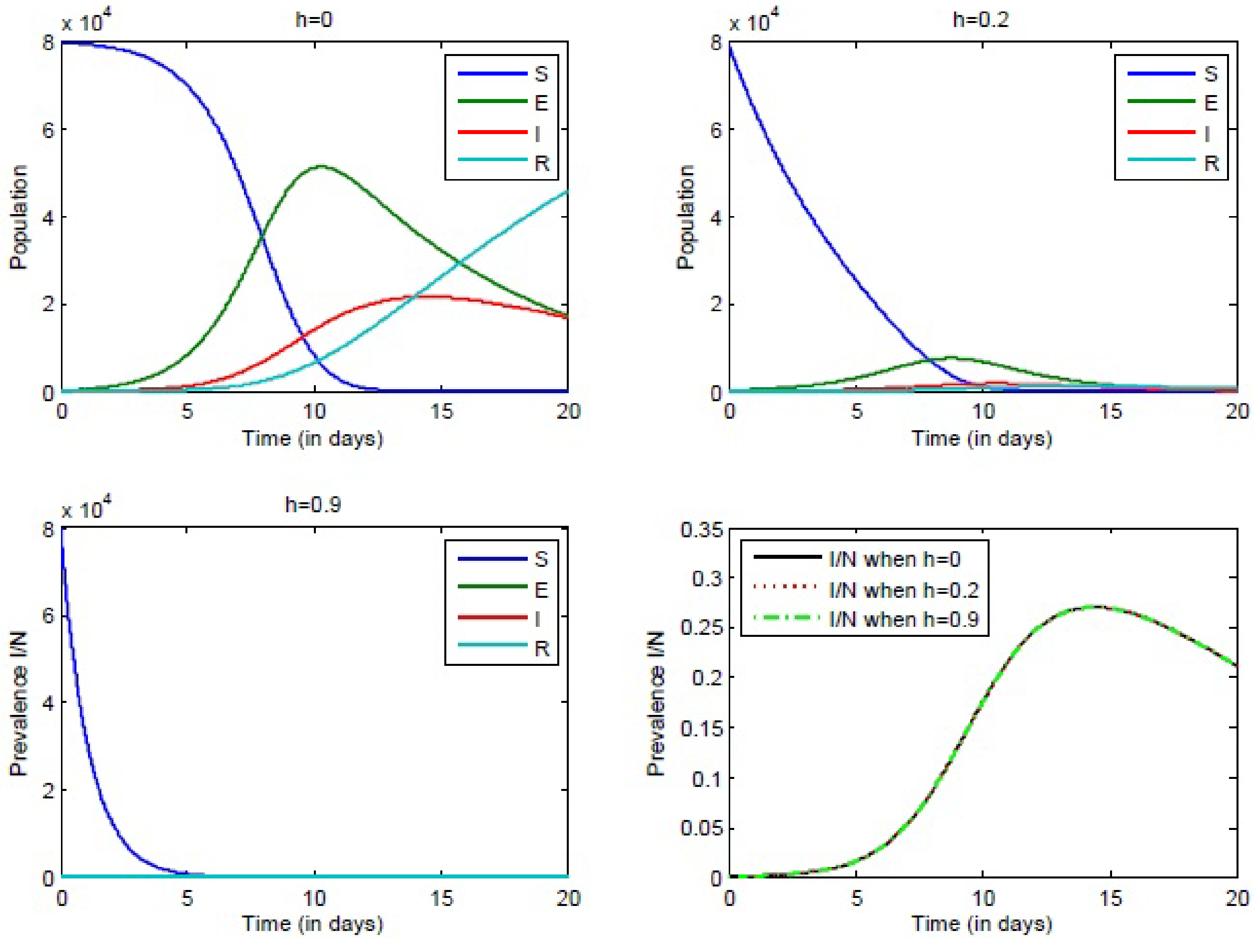
- The Hamiltonian and adjoint functions for the second objective function with both D.I. and D.D. are calculated the same way.

Model Simulations and Results: Case One

- **Case 1:** The 1st objective functional with density dependent birth and death terms.

$$J = \min \int_0^T \left(\frac{I(t)}{S(t)+E(t)+I(t)+R(t)} + B_1 h^2(t) \right) dt$$

Model Simulations and Results: Case One

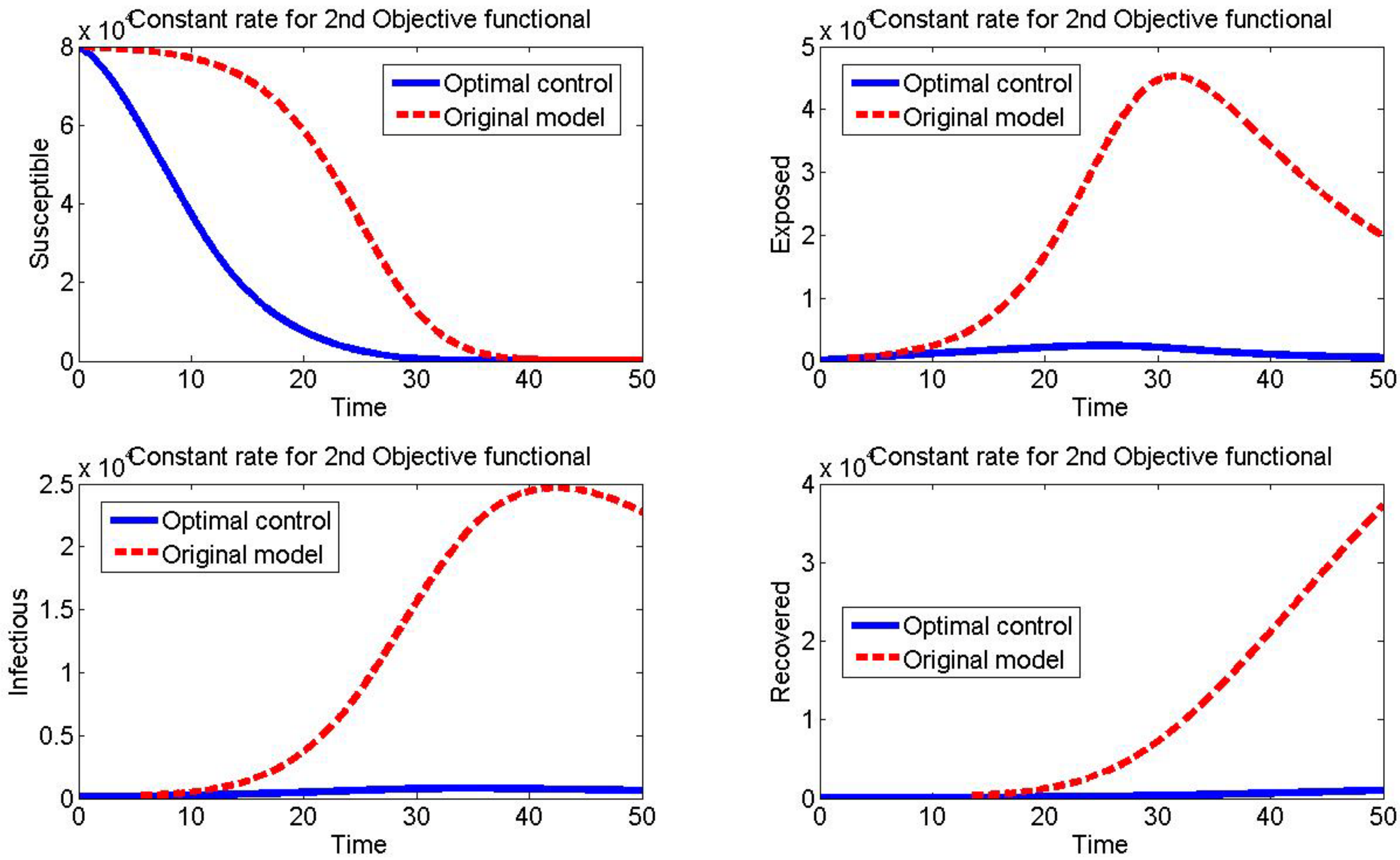


Model Simulations and Results: Case Two

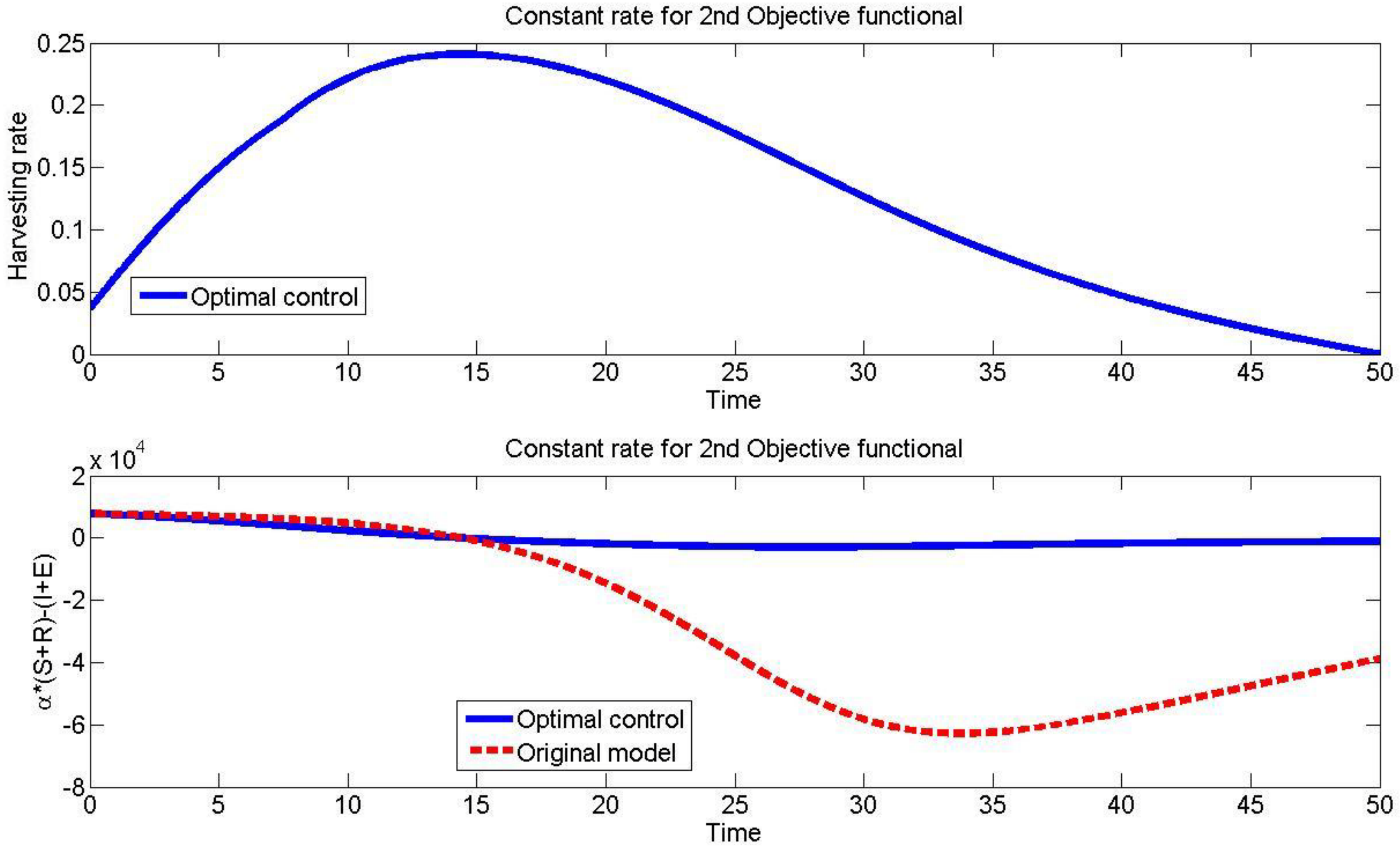
- **Case 2:** The 2nd objective functional with:
  a) density independent birth and death terms.
  b) density dependent birth and death terms.

$$J = \max \int_0^T \left(\alpha(S+R) - (E+I) - B_2 h^2(t) \right) dt$$

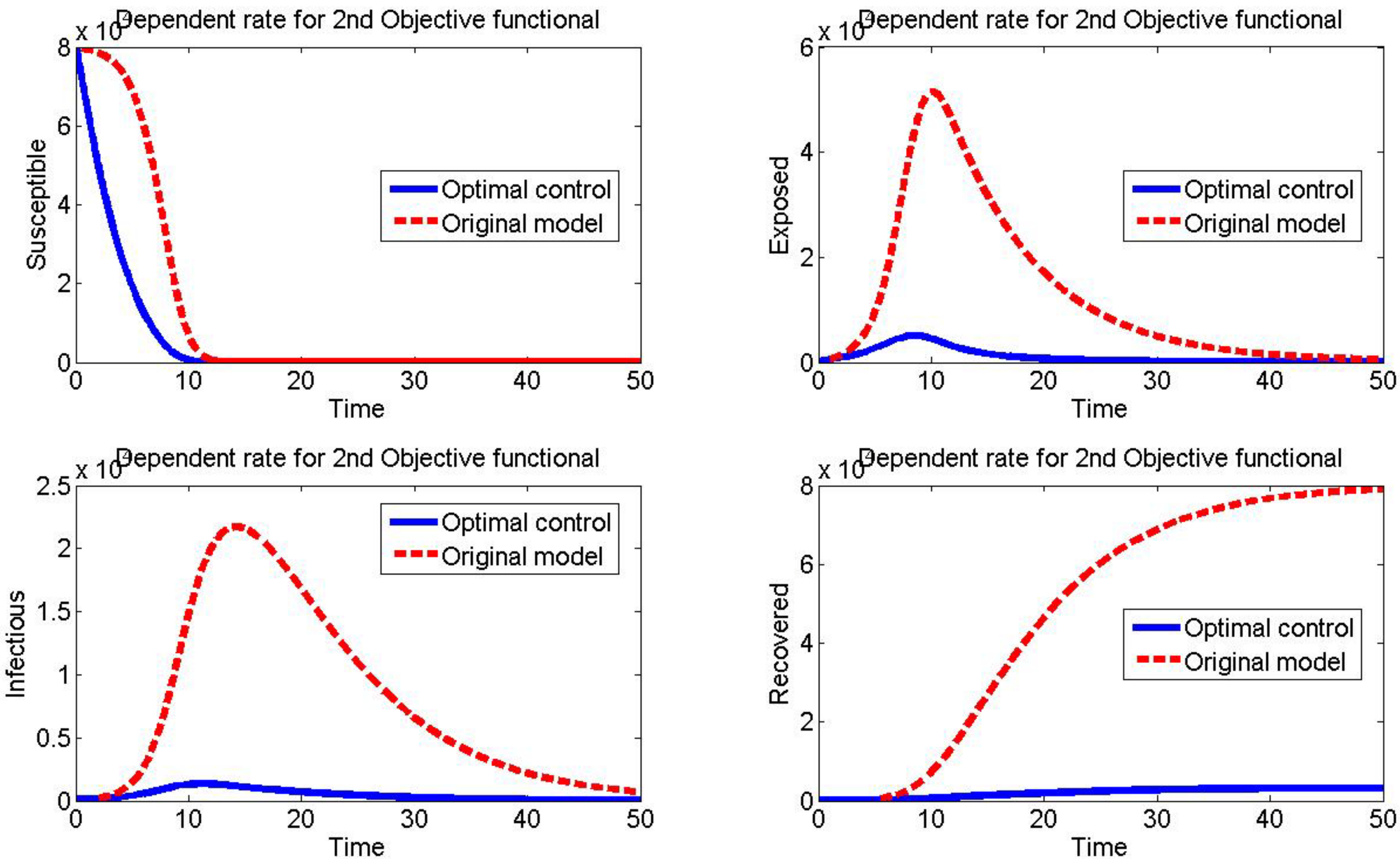
Model Simulations and Results: Case Two (a) D. I.



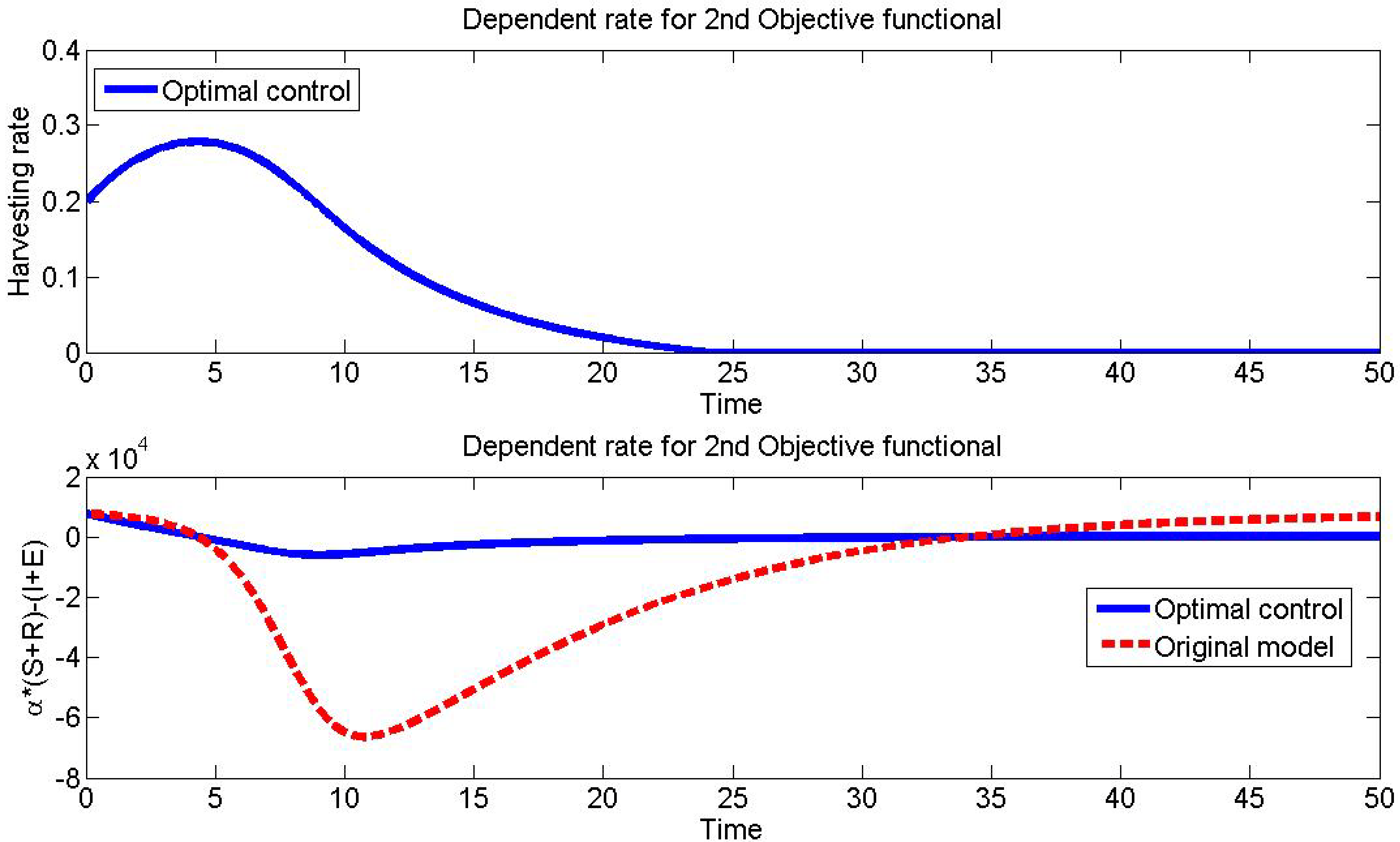
Model Simulations and Results: Case Two (a) D. I.



Model Simulations and Results: Case Two (b) D. D.



Model Simulations and Results: Case Two (b) D. D.



Conclusion

- Control has no effect on prevalence.
- Control aids the reduction of both exposed and infected hosts.
- Model Limitations and improvements.

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- Thank you for your attention!
- Questions?